THE QUANTUM COLLAPSE AND THE BIRTH
OF A NEW UNIVERSE

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Abstract

   The gravitational collapse and the birth of a new universe are
considered in terms of quantum mechanics. Transitions from annihila-
tion of matter to deflation in the collapse and from inflation to creation
of matter in the birth of a universe are considered. The creation of a
new universe takes place in another space-time since beyond the event
horizon the time coordinate is inextensible for an external observer. A
reasonable probability of this creation is obtainable only for miniholes.
The major part of the mass of such collapsing compact objects as stars,
quasars and active nuclei of galaxies remains confined in the potential
well near the vacuum state.

1 Introduction

The gravitational collapse problem is closely related to the cosmological prob-
lem. This has first been revealed in [1] where the geometry of a collapsing
body in the comoving coordinates was proved to be a geometry of Fried-
mann's closed world. At the same time it is a quantum consideration of
both the late collapse and the early Universe that is required for solving the
above-mentioned problems.

   It is also of interest to consider quantum-mechanically a birth of the uni-
verse in the laboratory [2, 3]. Some people consider a classical transition
from the collapse to the birth of a new universe [4, 5] and an eternal non-
singular black holes as a final stage of the collapse [6].

   In the present paper we use the approach introduced in [7, 8]. The
collapse in our universe is assumed to give rise to the birth of a new one.
From the viewpoint of quantum mechanics this is a tunnelling. In our space-
time the collapse ends in approaching Schwarzschild's horizon, i.e its final
stage is unobservable. The new universe is being born in another space-
time.

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2 Approach

In case the scale factor a and the scalar field ϕ are chosen as independent variables, Wheeler-DeWitt's equation in minisuperspace has the form [9]

$$[-\frac{1}{a^p}\frac{\partial}{\partial a}a^p\frac{\partial}{\partial a} + \frac{1}{a^2}\frac{\partial^2}{\partial\phi^2} + V(a,\phi)]\psi = 0 \tag{1}$$

where p takes account of the operator ordering (below we assume $p = 0$). Comparing (1) with Klein-Gordon's equation

$$-c^2\triangle\psi + \frac{\partial^2\psi}{\partial t^2} + \frac{m^2c^4}{\hbar^2}\psi = 0, \tag{2}$$

it is easy to see that in the Lorentzian domain $(V < 0)$ a plays the role of time and $\int a\,d\phi$ – the role of a coordinate, whereas in the Euclidean domain $(V > 0)$ $\int a\,d\phi$ plays the role of time and a – the role of a coordinate. For $\frac{\partial\psi}{\partial\phi} = 0$ (1) reduces to

$$-\frac{d^2\psi}{da^2} + V(a)\psi = 0. \tag{3}$$

The independence from ϕ is equivalent to homogeneity in the Lorentzian domain and stationarity in the Euclidean one. In Friedmann's universe the potential

$$V = \frac{2m_{pl}}{\hbar^2}[U(a) - E] \tag{4}$$

allows one to reduce (3) to Schrödinger's equation [7, 8], where

$$U(\gamma) = \frac{m_{pl}c^2}{2}(k\gamma^2 - B_0\gamma^4 - B_1\gamma^3 - B_2\gamma^2 - B_3\gamma - B_5\gamma^{-1} - B_6\gamma^{-2}), \tag{5}$$

k is the model parameter, $E = \frac{1}{2}m_{pl}c^2 B_4$, where $\gamma = \frac{a}{l_{pl}}$, B_n are the coefficients of the expansion of the energy density in Laurent's series in γ:

$$\varepsilon = \varepsilon_{pl}\sum_{n=0}^{6}B_n\gamma^{-n}, \tag{6}$$

$$n = 3(1 + \alpha), \tag{7}$$

where α is a coefficient in the equation of state $p = \alpha\varepsilon$ ($n = 0$ vacuum, $n = 1$ domain walls, $n = 2$ strings, $n = 3$ dust, $n = 4$ ultrarelativistic gas, $n = 5$ perfect gas, $n = 6$ ultrastiff matter).

Eq. (3) and formulae (4)–(7) describe the universe behaving as a planckeon with the energy of an ultrarelativistic gas $E = \frac{1}{2}m_{pl}c^2 B_4$ moving in the field of the rest matter ($B_n \neq B_4$).

3 Quantum Collapse

Since the geometry of a collapsing body in the comoving coordinates mimics Friedmann's closed model, the form of the potential in Schrödinger's equation should be the same. However, there arises some difference due to the equation of state of the collapsing body undergoing a sudden change from $p = \frac{\varepsilon}{3}$, $p = \frac{2}{3}\varepsilon$, $p = \varepsilon$ (annihilation of matter[1])to $p = -\varepsilon$ (deflation). This can be taken into account in modifying (5) by adding the terms with negative powers of $\gamma - \gamma_0$, where γ_0 corresponds to the scale factor at which the equation of state undergoes the change. Finally the potential takes the form

$$U(\gamma) = \frac{m_{pl}c^2}{2}[k\gamma^2 - B_0\gamma^4 - B_1\gamma^3 - B_2\gamma^2 - B_3\gamma - B_5\gamma^{-1} - B_6\gamma^{-2}$$

$$-B_5'(\gamma - \gamma_0)^{-1} - B_6'(\gamma - \gamma_0)^{-2}], \tag{8}$$

where $k = +1$.

For $\gamma \gg \sqrt[4]{\frac{2E}{m_{pl}c^2}} \gg 1$ and $B_0 = 1$ Eq. (3) reduces to

$$\frac{d^2\psi}{d\gamma^2} + \gamma^4\psi = 0, \tag{9}$$

which has a solution   [10]

$$\psi = \gamma^{\frac{1}{2}} Z_{\frac{1}{6}}(\frac{\gamma^3}{3}). \tag{10}$$

with the asymptotics   [9]

$$\psi = C_1 e^{i\frac{\gamma^3}{3}} + C_2 e^{-i\frac{\gamma^3}{3}}. \tag{11}$$

This WKB formula corresponds to the classical inflation $\gamma = e^{t/t_{pl}}$ or deflation $\gamma = e^{-t/t_{pl}}$ since the action $S \propto (\pm a^3)$ and on the other hand $S = \int L\, d\eta$,

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[1]by annihilation of matter we mean not a process inverse to pair creation but that related to nonconservation of a baryon charge

where the Lagrangian $L \propto a^4$ and $d\eta = dt/a$ (η is the conformal time), hence $\dot{a} = \pm Ha$ and $a = Ce^{\pm Ht}$ with $H = 1/t_{pl}$.

For $\gamma - \gamma_0 \ll 1$ ($B_0 = 1$) Eq. (3) takes the form

$$\frac{d^2\psi}{d\gamma^2} + [\gamma_0{}^4 + B_5'(\gamma - \gamma_0)^{-1} + B_6'(\gamma - \gamma_0)^{-2} + \frac{2E}{m_{pl}c^2}]\psi = 0, \qquad (12)$$

which has a solution   [11]

$$\psi = \text{const} \times \rho^{s+1} e^{-\frac{\rho}{2}} F(-p, 2s + 2, \rho), \qquad (13)$$

satisfying the boundary condition $\psi(\gamma_0) = 0$,
where F is a degenerate hypergeometric function,

$$\rho = 2(\gamma - \gamma_0)\sqrt{-\gamma_0{}^4 - \frac{2E}{m_{pl}c^2}},$$

$$n = \frac{B_5'}{2\sqrt{-\gamma_0{}^4 - \frac{2E}{m_{pl}c^2}}} \quad (B_5' > 0),$$

$$s = -\frac{1}{2} + \sqrt{\frac{1}{4} - B_6'},$$

$$n - s - 1 = p = 0, 1, 2, ...$$

The energy spectrum has the form

$$E_p = -\frac{B_5'{}^2 m_{pl}c^2}{8(p + \frac{1}{2} + \sqrt{\frac{1}{4} - B_6'})^2} - \frac{\gamma_0{}^4}{2} m_{pl}c^2. \qquad (14)$$

It should be noted that this discrete spectrum does exist only if the conditions

$$\gamma_0{}^4 \leq \frac{2|E|}{m_{pl}c^2} \leq \frac{2M}{m_{pl}} \qquad (15)$$

and

$$B_6' \leq \frac{1}{4} \qquad (16)$$

are valid, where M is the rest mass of a collapsing body. As seen from (8), (15) results in the absolute value of the vacuum energy not exceeding $|E|$.

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The last equality in (15) is obtainable if one assumes $E + Mc^2 = m_{pl}c^2$, where $M \gg m_{pl}$.

The compact objects are known to collapse for $M = 10 - 10^9 M_\odot$: stars at $M \sim 10 M_\odot$ and quasars and active nuclei of galaxies at $M \sim 10^9 M_\odot$. Hence

$$\gamma_0 \leq \sqrt[4]{\frac{2M}{m_{pl}}} = \sqrt[4]{\frac{r_g}{l_{pl}}},$$

i.e. $\gamma_0 \leq 10^{10}$ or $a_0 \leq 10^{-23}$cm for stars and $\gamma_0 \leq 10^{12}$ or $a_0 \leq 10^{-21}$cm for quasars and active nuclei of galaxies.

Eq. (12) reduces to Eq. (9) at $\gamma = \gamma_0 + 1$ for $\gamma_0 \gg 1$ if $B_5' + B_6' + B_4 = 0$, which results in $B_5' \approx= -B_4$ since $B_6' \leq \frac{1}{4} \ll |B_4|$ ($B_6' \geq 0$). Assuming $B_5' = -B_4$, $B_6' = 0$, we obtain

$$B_4 = -2n^2 + 2n\sqrt{n^2 - \gamma_0{}^4} \tag{17}$$

where $B_4 = -2\gamma_0{}^4$ at $n = \gamma_0{}^2$ and $B_4 = -\gamma_0{}^4$ for $n \gg \gamma_0{}^2$. This means that only higher levels are possible for $\gamma_0 \gg 1$.

From (6) it follows that the absolute value of the partial energy density $|\varepsilon_4| = \varepsilon_{pl} \frac{B_4}{\gamma_0{}^4} \sim \varepsilon_{pl}$ at $\gamma = \gamma_0$ since $\frac{|B_4|}{\gamma_0{}^4} \sim 1$.

For γ very close to γ_0, when $\gamma - \gamma_0 \ll \sqrt{\frac{B_6' m_{pl} c^2}{2E}}$, Eq. (12) reduces to

$$\frac{d^2\psi}{d\gamma^2} + \frac{B_6'}{(\gamma - \gamma_0)^2}\psi = 0, \tag{18}$$

which has a solution   [10]

$$\psi = \sqrt{\gamma - \gamma_0} \begin{cases} C_1 \cos[(b\ln(\gamma - \gamma_0)] + C_2 \sin[b\ln(\gamma - \gamma_0)], & b^2 = B_6' - \frac{1}{4} > 0; \\ C_1(\gamma - \gamma_0)^b + C_2(\gamma - \gamma_0)^{-b}, & b^2 = \frac{1}{4} - B_6' > 0; \\ C_1 + C_2 \ln(\gamma - \gamma_0), & B_6' = \frac{1}{4}. \end{cases}$$
$$\tag{19}$$

For B_6' there occurs a fall to the field centre, which corresponds to $E_0 = \infty$ [11].

4   Birth of a Universe

After the tunnelling through a potential barrier the collapsing body appears in the potential well where for $\gamma \ll 1$ Schrödinger's equation reduces to

[7, 8]

$$\frac{d^2\psi}{d\gamma^2} + \left(\frac{B_5}{\gamma} + \frac{B_6}{\gamma^2} + \frac{2E}{m_{pl}c^2}\right)\psi = 0 \tag{20}$$

whose solution is given by formula (13), satisfying the boundary condition
$\psi(0) = 0$,
where

$$\rho = 2\gamma\sqrt{\frac{-2E}{m_{pl}c^2}},$$

$$n = \frac{B_5}{2\sqrt{\frac{-2E}{m_{pl}c^2}}}.$$

The energy spectrum has the form

$$E_p = -\frac{B_5{}^2 m_{pl}c^2}{8\left(p + \frac{1}{2} + \sqrt{\frac{1}{4} - B_6}\right)}. \tag{21}$$

For very small γ, when $\gamma \ll \sqrt{\frac{B_6 m_{pl}}{2E}}$, ψ -function has the form (19) with
$\gamma - \gamma_0$ substituted by γ and B_6' by B_6. The WKB coefficient for penetration
through the potential barrier at $a > 0$ reads [11]

$$D = \exp\left[-\frac{2l_{pl}}{\hbar} \left| \int_{\gamma_1}^{\gamma_2} \sqrt{2m_{pl}(E - U)}\, d\gamma \right|\right] \tag{22}$$

where $\gamma_1 \approx \sqrt[4]{-B_4}$, $\gamma_2 \approx -\frac{B_5}{B_4}$, $E = \frac{1}{2}m_{pl}c^2 B_4$. U is given by formula (8) with
$B_5' = B_6' = 0$ since we consider tunnelling in the domain where $\gamma < \gamma_0$. For
$|B_4| \gg 1$ we obtain

$$D \approx e^{-2\sqrt[4]{(-B_4)^3}}. \tag{23}$$

According to (14) $|B_4| \leq \frac{2M}{m_{pl}}$. Hence we obtain $D \geq e^{-10^{29}}$ for stars, $D \geq$
$e^{-10^{35}}$ for quasars and active nuclei of galaxies.
The penetration factor given by formula (22) has first been calculated by G.
Gamow [12] for the case of alpha decay in radioactive nuclei. Gamow's
procedure was extended in [13, 14, 15, 16, 17, 18, 9, 19] to the case of the
birth of the Universe from a pure vacuum.

The probability of the birth of a new universe, as a result of the gravi-
tational collapse in our space, is $W = D^2$ due to penetration through two
barriers: at $a > 0$ in our space-time and at $a < 0$ in another space-time

where the birth takes place. For the sake of simplicity, we may identify the other space-time with the negative semiaxis of the scale factor. The universe cannot be born in the space-time where the gravitational collapse occurs, because for an external observer the time coordinate is inextensible beyond the event horizon, i.e. through the point with $t = +\infty$. Then the scenario in the new universe (other space-time) might be a mirror reflexion of that in our space-time. The total scenario for $-\infty < \gamma < +\infty$ is determined by the potential $U(|\gamma|)$. Near the singularity at $\gamma = 0$ there will be twice as many energetic levels as given by formula (14). For the new universe at $\gamma \approx -\gamma_0$ there occurs a transition from the equation of state $p = -\varepsilon$ (inflation) to $p = \frac{\varepsilon}{3}$, $p = \frac{2}{3}\varepsilon$, $p = \varepsilon$ (creation of matter). The same transition takes place in our universe at $\gamma \approx \gamma_0$ (reheating after inflation   [19]).

To create our universe with the mass $M = 10^{55}$g   [9] (if it is assumed to be closed), it is neccesary for the parameter $\gamma_0 = e^{t/t_{pl}}$ to be equal to 10^{15}. Hence the time of the end of inflation is $t = 35t_{pl}$, which agrees with classical estimates within a factor of two   [9, 19]. The WKB penetration factor $D \geq e^{-10^{45}}$ for our universe, the probability of its creation $W \geq e^{-2\cdot10^{45}}$.

The infintesimal values of the penetration factor for collapsing stars, galaxies, quasars or for the birth of our universe mean that not all mass of these objects, may be a small part of it, tunnels. The major part of the mass of these objects is confined in the potential well near $\gamma = \gamma_0$, whose left boundary corresponds to the vacuum state, never reaching the singularity at $\gamma = 0$.

If, e.g., we assume that $\gamma_0 = 10$, $B_4 = -10^4$ and $M = 5 \cdot 10^3$, then $D = e^{-2\cdot10^3}$. Thus, a reasonable penetration factor is obtainable only for miniholes. As to Hawking's effect, it is easy to see that the evaporation time $\tau_H = \frac{Mc^2}{P_H}$, where $P_H = \frac{\hbar\kappa^2}{960\pi c^2}$ [20], reads

$$\tau_H = 1.536 \cdot 10^4 (\frac{M}{m_{pl}})^3 t_{pl}. \tag{24}$$

Whereas the deflation time

$$t_d = \frac{t_{pl}}{4} \ln \frac{2M}{m_{pl}}. \tag{25}$$

Thus $t_d \ll \tau_H$ for $M > m_{pl}$. Hence Hawking's effect is negligible as compared with the deflation. Apropos, from (25) it follows that the minimum collapsing mass $M = \frac{1}{2}m_{pl}$.

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5   Conclusion

We have considered the total cycle from the quantum gravitational collapse of a body to the birth of a new universe in another space-time. However, the tunnelling that accompanies the collapse occurs with an extremely low probability, which makes us assume that not all mass of the collapsing compact object, such as a star or a galaxy, tunnels through the barriers separating it from the singularity or another space-time where a new universe emerges. The major part of the mass of these objects remains confined in the potential well near the vacuum state. The tunnelling giving rise to a new universe may occur only for miniholes.

6   Acknowlegement

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